Summary
The basic question to answer here is, which level of detectability and isolability is possible to achieve based on a given process model. This question is answered without designing a diagnosis system, it is a pure model property.

Model Form
The model is assumed to be in the form of a set of differential-algebraic equations where different subsets of equations are valid for different behavioral modes.

Example. A model of a battery with two modes OK and Deg:

Equations
\[ \begin{align*}
& v = u_0 - R \cdot i \\
& T = \alpha(T_{\text{amb}} - T) + \beta i^2 R \\
& \text{OK}(B) \rightarrow R = R_{\text{nom}} \\
& \text{Deg}(B) \rightarrow R = 0
\end{align*} \]

\[ B \text{ is the mode of the battery; } v, R, i, \text{ and } T \text{ internal unknown variables; } u_0, R_{\text{nom}}, \text{ and } T_{\text{amb}} \text{ known constants. The equations (1)-(3) describes the behavior in the no-fault mode.} \]

Generally, let the set of equations implied by mode \( b_i \) be denoted by

\[ M_b(x, x, z) = 0 \]

where \( x \) are internal unknown variables, \( z \) known variables. Mode \( b_i \) is either the no-fault mode \( b_0 = \text{NF} \) or a fault \( b_i = f_i \).

Basic Definitions

Definition 1 (Observation set).

\[ \mathcal{O}(b_i) = \{ z| \exists x. M_b(x, x, z, 0) = 0 \} \]

The observation set is the set of all possible observations in a particular mode. Based on this, detectability and isolability are defined as:

Definition 2 (Detectability).

A fault \( f_i \) is said to be detectable if

\[ \mathcal{O}(f) \nsubseteq \mathcal{O}(\text{NF}) \]

Definition 3 (Isolability).

A fault \( f_i \) is said to be isolable from fault \( f_j \) if

\[ \mathcal{O}(f_i) \nsubseteq \mathcal{O}(f_j) \]

Thus, a fault is detectable if there exists any fault situation that produces observations not consistent with the nominal model and correspondingly for the isolability requirement.

Diagnosability - Analytic

For linear DAEs, the observation sets can explicitly be expressed only in \( z \) by straightforward elimination of \( x \).

Theorem 1. Let \( \mathcal{O}(b_i) = \{ z|R_i(p)z = 0 \} \), then mode \( b_i \) is not isolable from mode \( b_j \) if and only if

\[ \forall s \in \mathbb{C}. \text{rank} \left( R_i(s) \right) = \text{rank} \left( R_j(s) \right) \]

Diagnosability - Structural

For non-linear systems, structural methods can be used.

The figure shows the Dulmage-Mendelsohn’s (DM) decomposition of the structure of \( M_b \). Blue area indicates the part of the model that contains redundancy, i.e., the equations in \( M_b \) that can be tested.

Theorem 2. \( b_i \) is not isolable from \( b_j \) if \( M_{b_j}^+ \subseteq M_{b_i}^+ \)

Intuition: \( M_{b_j}^+ \subseteq M_{b_i}^+ \Rightarrow \mathcal{O}(b_i) \nsubseteq \mathcal{O}(b_j) \)

Example - Diesel Engine Model

Model properties: 528 variables (8 states), 532 equations, 11 known variables, 7 faults.

Results: Structural analysis provided the following results in 0.17 seconds on a standard PC:

- The horizontal lines indicate equations related to faults.
- The red rectangles are blocks in the DM-decomposition.
- Faults affecting the same grey block are not isolable.