Contributions
- Parameter estimation in mixed linear/nonlinear state-space models.
- An extension of a Rao-Blackwellized particle smoother, capable of handling fully interconnected mixed linear/nonlinear models.

Problem Formulation

The problem is to identify the parameters $\theta$ in a mixed linear/nonlinear state space model

$$
\begin{align*}
\mathbf{a}_{t+1} &= f_{a}(\mathbf{a}_t, u_t, \theta) + A_a(\mathbf{a}_t, u_t, \theta)\mathbf{z}_t + w_{a,t}, \\
\mathbf{z}_{t+1} &= f_{z}(\mathbf{a}_t, u_t, \theta) + A_z(\mathbf{a}_t, u_t, \theta)\mathbf{z}_t + w_{z,t}, \\
y_t &= h(\mathbf{a}_t, u_t, \theta) + C(\mathbf{a}_t, u_t, \theta)\mathbf{z}_t + e_t,
\end{align*}
$$

(1)

using maximum likelihood (ML) estimation.

EM Algorithm

Expectation Maximization (EM) is used to compute the ML estimate

$$\hat{\theta} = \arg \max_{\theta} p_{\theta}(y_1, \ldots, y_N).$$

The E Step of the algorithm contains intractable expectations, which are computed approximately using Particle Smoothing (PS).

\begin{align*}
\text{(E Step): } & \text{Calculate} \\
& Q(\theta, \theta_k) = E_{\theta_k} \{ \log p_{\theta}(X_N, Y_N) \mid Y_N \} \\
\text{(M Step): } & \text{Solve} \\
& \theta_{k+1} = \arg \max_{\theta} Q(\theta, \theta_k)
\end{align*}

Rao-Blackwellized PS

The Monte Carlo variance of the PS can be reduced by exploiting the structure in (1), leading to a Rao-Blackwellized PS (RBPS).

Previous RBPS only apply to model (1) in special cases ($A_a \equiv 0$). Hence, an existing RBPS has been extended to handle the fully interconnected model (1) under study.

Experimental Results

The proposed method, based on EM and a new RBPS, is compared with a similar method based on EM and a standard PS. The results from a four-dimensional system ($\dim a_t = 1, \dim z_t = 3$) with one unknown parameter are given in Fig. 1.

Conclusion

Through simulations it has been shown that using a RBPS instead of a PS reduces the variance, not only of the estimated states, but also of the estimated parameters.

Fig. 1. Parameter estimates vs. iteration number for RBPS-EM (left) and PS-EM (right).